RESEARCH STATEMENT OF VERA VÉRTESI

Most of the topics I have researched can be associated with the following two topics: algebraic complexity and universal algebra. I give a brief description of each topic.

ALGEBRAIC COMPLEXITY

The *identity checking problem* asks whether two given terms over an algebra agree for every substitution. For a certain kind of structures (like groups, rings, semigroups, etc.) usually we are interested in verifying that some kind of duality holds. That is, the identity checking problem is easy for some algebras and hard for the rest.

For RINGS the question is solved. H. Hunt and R. Stearns proved that the identity checking problem is in P if the ring is nilpotent and coNP-complete otherwise. Burris and Lawrence proved that the same holds for rings, in general. So, for example, for the ring $\mathbb{Z}_2$, the coNP-completeness of the identity checking problem is an easy consequence of the NP-completeness of 3-SAT. But the proof uses high powers of sums. This is the reason why J. Lawrence and R. Willard introduced the $_\Sigma$ version of the problems. The problem remains the same, but the form of the input is restricted to those terms that are sum of monomials. J. Lawrence and R. Willard had previously shown the coNP-completeness of the $_\Sigma$ version of the identity checking problem for matrix rings large enough to have a nonsolvable group of units. In 2004 with Cs. Szabó [1, 2] we proved—by using only "ordinary" semigroup words—that the identity checking problem is coNP-complete for all other matrix semigroups: $M_2(\mathbb{Z}_2)$ and $M_2(\mathbb{Z}_3)$. In [3] we prove that the duality holds for the $_\Sigma$ version as well, it is polinomial, if the ring modulo its Jacobson-radical is commutative, and coNP-complete otherwise.

The answer for SEMIGROUPS is less complete. Till 2002 there was not known any semigroup for which the identity checking problem is hard. V. Popov and M. Volkov exhibit a semigroup of size $\leq 2^{1700}$ with a coNP-complete identity checking problem. Later Kisieliewicz presented an example of a few hundred size. Recently with Cs. Szabó [1] we constructed a semigroup of size 13 with coNP-complete identity checking problem. The smallest example of coNP-complete identity checking problem is a 6-element semigroup, the Brandt-monoid, given by Klima. As it is proved by S. Seif, Cs. Szabó for 0-simple semigroups the identity checking problem is in P when the structure group is trivial (i.e., for combinatorial 0-simple semigroups). For other kind of 0-simple semigroups the problem can be hard, for example, if the identity checking problem is coNP-complete for the structure group, then it is coNP-complete for the semigroup itself. There has not been known any 0-simple semigroup with a structure group for which the identity checking problem is in P, but still having a coNP-complete identity checking problem. In [4] we show such an example with 19 elements.

UNIVERSAL ALGEBRA

The *membership problem* asks whether a finite algebra belongs to the variety generated by a given finite algebra. As varieties are equational classes, this problem can be checked by equation testing. In some sense the *β-function* or *equational bound*, defined by G. McNulty is the measure of the complexity of the membership problem. $\beta(n)$ is the minimum length of such equations that are sufficient to check to determine whether an n-element algebra is in the variety or not. This function exists and it is uniquely determined for any variety, and it is recursive (it can be algorithmically computed). The question, which is investigated by many universal algebraists (e.g. G. McKenzie, R. Willard) is: what is the order of magnitude of this function. By the construction of the free-algebra one can show that the β-function is at most triple-exponential. Z. Székely found an algebra for which the β-function is sublinear. In [5] we investigate the β-function for hypergraph algebras, and prove that in general it is not bounded by any polynomial.

References

[1] Cs. Szabó and Vera Vértesi. The complexity of the word-problem for finite matrix rings. *Proceedings of the American Mathematical Society*, 132:3689–3695, 2004.

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[3] Cs. Szabó and Vera Vértesi. The identity checking problem in finite rings. manuscript.

[4] M. Volkov S. Pletseva and Vera Vértesi. The complexity of the identity checking problem in 0-simple semigroups. manuscript.

[5] G. Kun and Vera Vértesi. The membership problem in finite flat hypergraph algebras. *International Journal of Algebra and Computation*. submitted.